Evolutionary Optimization of a Multi-location Inventory Model with Lateral Transshipments

by

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1. Introduction

The most problems in inventory management have a difficult and complex structure without any recognizable function which is optimizable by analytic algorithms. That is the same like in a lot of other logistic, transport or production related fields. In some special cases we know parts of the objective function or some characteristics of the systems behaviour by given parameters. We can find global strategies for ordering, stocking levels or shipment by investigation of models with many assumptions. These assumptions lead not seldom to a strange limitation of the models' validity. Therefore, my question was: Is there any other way to solve such problems without limiting assumptions and strenuous analytical investigations?
The optimization of inventories, production systems, transport and logistic problems means at present: An integral combination of quick simulators with efficient optimization tools. The classic optimization methods like the gradient method, the dynamic programming or any kind of heuristics come across limits very often concerning runtime and complexity. Therefore, we need optimization algorithms, which change automatically the input of parameters for the next simulation run by considering the user defined objectives and the results of the past runs.
Evolutionary algorithms are an optimization methodology based on a direct analogy to Darwinian natural selection, recombination and mutation in biological reproduction. The evolution process thus converges to individuals with an optimal fitness to the considered environment. The main advantages of these algorithms lie in great robustness, problem independence and high parallel working. So far, evolutionary algorithms were most successful in parameter optimization domains. However, even there are certain problems, as lack of final tuning capabilities and severe time complexity, prohibit their wider use on most moderately and highly complex problems.
Evolution strategies are a subset of evolutionary algorithms. They provide a powerful solving of complex problems with more or less continuously changeable parameters, e.g. current optimization, vector or parameter optimization in general. The aim of my work was to investigate the applicability of evolutionary algorithms to the optimization of stocking levels and inventory costs in general. The problem to find an optimal control for a multi-location inventory model with lateral transshipments in general can not be solved in an analytical way. The main reason for having only a few results on multi-location models with lateral transshipments is the fact that the solution of such models is connected with a lot of analytical and numerical problems. In the case of non-linear cost functions until now no analytical solutions exist, and they will hardly be expected. Thus corresponding investigations should be concentrated on algorithmic solutions and/or various search procedures. One very promising attempt in this direction is the application of evolutionary algorithms. Since the last decade the literature on evolutionary algorithms has been growing. For an insight into base methods and

main fields of applications see e.g. [18], [3], [5] or [22] for current problems and applications. However, in fact there are no applications to inventory models, especially to multi-location models with transshipments. A first attempt in that direction was made by ARNOLD in [2].
The present paper is devoted to the search of order decisions, which are optimal or at least sub-optimal for the N-location-infinite-period model with transshipments. The investigated model is described in the next section, and the main results will be outlined. Basic notions and concepts of evolutionary optimization are given in Section 3. Section 4 contains the evolutionary optimization of the considered multi-location model for 4 and 5 locations. Some numerical examples show the practicability and give an idea of the performance of the proposed approach. A short summary and an outlook for further investigations are given in the final Section 5.

2. The Multi-location Inventory Model with Transshipments

The investigation of N-location models with lateral transshipments, $N \geq 2$, is an important problem for mathematical inventory theory as well as for inventory practice. The combination of N locations can result in a system with vertical, horizontal, and mixed structure. The classical Echelon-models (cp. [4]) belong to the systems with vertical structure. While Echelon-models are widely investigated the results on systems with horizontal structure are smaller. The two-location-one-period case with linear cost functions is considered by AGGARWAL [1] and KÖCHEL [10]. KRISHNAN and RAO [14] deal with a N-location-one-period model, where the cost parameters are the same for all locations. An approximate solution for the N-location-one-period model is given by KÖCHEL [11]. KÖCHEL [12] investigates for the first time the N-location-infinite-period model with linear cost functions. A dynamic model with finite horizon is examined by ROBINSON [16]. The effect of lateral transshipment on the service levels in a two-location-one-period model is studied by TAGARAS [21]. In all works no fixed costs are assumed. Some results on a two-location-one-period model with fixed order costs are given by HERER and RASHIT [7].
In KÖCHEL [12] the following inventory model is considered: There are N locations which are all storing the same product. At the beginning of a period t =1, 2, ... additional inventory can be ordered by an order decision (OD). Ordered inventory is received immediately by cost $k_i > 0$ for one unit in location i. During period t demand occurs in accordance with a random vector $\underline{s}(t) = (\underline{s}_1(t), ..., \underline{s}_N(t))$. It is assumed that $\{\underline{s}(t);\ t=1, 2,...\}$ is a sequence of independent identically distributed random vectors. Let $E[\ \underline{s}_i(t)\ ] = \mu_i$ exist with $0 < \mu_i < \infty$ for i=1(1)N. At the end of a period it is possible to redistribute the present stock by a transportation decision (TD). Transshipment occurs immediately by cost $c_{ij} > 0$ for one unit transported from location i to location j. After this the unsatisfied demand is backlogged and costs are incurred - holding cost $h_i > 0$ per unit of undemanded inventory or shortage cost $p_i > 0$ per unit of unsatisfied demand in location i. The problem is to find a policy (i.e., a sequence of OD´s and TD´s) which minimizes the expected average cost over an infinite horizon. Such a policy is called average optimal.
To fix somewhat narrower bounds for the optimization problem the following assumptions concerning the cost parameters are introduced in [12]:

(A.1) $\quad h_i + p_j - c_{ij} \geq k_i - k_j$;
(A.2) $\quad h_j - h_i + c_{ij} \geq k_j - k_i$ and $\quad p_i - p_j + c_{ij} \geq k_j - k_i$;
(A.3) $\quad k_i + c_{ij} \geq k_j$ and $\quad c_{ik} + c_{kj} \geq c_{ij}$, i, j, k =1(1)N, $i \neq j \neq k$.

These assumptions result from the background of the problem, and they have a clear economic interpretation. Assumption (A.1) means the *effectiveness of transshipments* - the reward in the present period from the transport of one inventory unit from location i with positive rest stock to location j with negative rest stock is not smaller than the corresponding alteration of order cost in the following period. The *relative independence of all locations* is expressed by assumption (A.2) - it is not profitable to redistribute an inventory unit between locations with positive rest stocks as well as an unsatisfied demand unit between locations with negative rest stocks. Assumption (A.3) stands for *shortest way conditions* - it is more expensive to order and to transship via another location than by the direct route. With assumptions (A.1) to (A.3) the considered optimization problem is solved in [12] and [13]. To report the main results I denote the inventory level after an OD by $\mathbf{a} = (a_1, a_2, ..., a_N)$ and the vector of rest stock after occurring realization $\mathbf{s} = (s_1, s_2, ..., s_N)$ of the demand by $\mathbf{y} = \mathbf{a}-\mathbf{s}$. Transshipment is organized in accordance with TD $\mathbf{B} = (b_{ij})$ with $b_{ij} \geq 0$ as the transported quantity from location i to location j; $i, j = 1(1)N$.

Property 1 (cp. Satz 3.1 in [12]) :
The set of stationary policies contains an average optimal policy, i.e., the optimization can be restricted to policies which use in all periods the same rules to choose the OD or TD respectively.
Let $g(\cdot)$ be a function defined for $\mathbf{a} \in R^N$ as

$$(1) \qquad g(\mathbf{a}) = \sum_{i=1}^{N} [\, k_i \mu_i + L_i(a_i) \,] - C(\mathbf{a}) \,,$$

where

$$(2) \qquad L_i(a_i) = E\,[\, h_i \max(0; a_i - \underline{s}_i) + p_i \max(0; \underline{s}_i - a_i) \,]$$

describes the expected holding and shortage cost in location i with stock level a_i after OD, $i=1(1)N$, and $C(\mathbf{a})$ is the expected system's gain from optimal TD (redistribution) if OD $\mathbf{a}$ was chosen.

Property 2 (cp. Lemma 4.2 in [12]) :
There exists a solution $\mathbf{a}^*$ of the convex problem $\min_{\mathbf{a} \in R^N} g(\mathbf{a})$, where $g(\cdot)$ is defined by (1).

Property 3 (cp. Theorem 2.2 in [13]):
(a) $\mathbf{a}^*$ is the OD in the stationary average optimal policy, i.e., the average optimal order policy is a $(\mathbf{S}, \mathbf{S})$ - policy with $\mathbf{S} = \mathbf{a}^*$.
(b) For any rest stock vector $\mathbf{y}$ the optimal TD $\mathbf{B}^* = (b_{ij}^*)$ is a solution of an open linear transporting problem with supplier set $I^+ = \{ i=1(1)N: y_i > 0 \}$, consumer set $I^- = \{ i=1(1)N: y_i < 0 \}$ and coefficients $C_{ij} = h_i + p_j - c_{ij} + k_j - k_i > 0$ for $i \in I^+$ and $j \in I^-$.

Since $C(\mathbf{a}) = E\,[\, C(\mathbf{a},\underline{\mathbf{s}}) \,]$ with $C(\mathbf{a},\mathbf{s}) = \sum_{i \in I^+} \sum_{j \in I^-} b_{ij} C_{ij}$ for $\mathbf{s} \geq (0, ..., 0)$,

from Property 3 follows that an analytical tractable expression for function $g(\cdot)$ exists only for $N = 2$ or if $C_{ij} = C > 0$ for $i, j = 1(1)N$, $i \neq j$. In both cases the open linear transporting problem in Property 3(b) has an analytical solution. In the general case there are at least two approaches - approximate solution and simulation. The approach with the approximate solution replaces the coefficients C_{ij} by the constant $\underline{C} = \min \{ C_{ij} : i, j=1(1)N, i \neq j \}$. This has a twofold effect. We have a lower bound $\underline{C}(\mathbf{a})$ for $C(\mathbf{a})$, and a general solution of the transporting problem is possible, i.e., $\underline{C}(\mathbf{a})$ is given in an analytical way for $\mathbf{a} \in R^N$. For more information see KÖCHEL [11]. The simulation approach combines simulation and optimization and uses the following procedure for defining nearly-optimal order decisions:

1. By simulation we generate a sample of demand realizations.
2. For given OD $\mathbf{a} \in R^N$ we compute the estimated $g(\hat{\mathbf{a}})$ for $g(\mathbf{a})$ defined in (1).
3. Since $g(\cdot)$ is convex for every sample we use one of the methods for minimizing a convex function to find OD $\mathbf{a}$ which realizes the minimum.
4. We choose OD $\hat{\mathbf{a}}$ as approximation for $\mathbf{a}^*$ and the value $g(\hat{\mathbf{a}})$ as approximation for $g(\mathbf{a}^*)$.

We remark, that this approach has the essential advantage that multi-location models with arbitrary distributed demand and even with dependencies of the demand between locations can be investigated. A crucial point for the effectiveness of the described procedure is that the function to be minimized is convex. On the other hand the convexity of function $g(.)$ is a consequence of the assumption that all cost functions in the multi-location inventory model are assumed to be linear without any fixed cost factor. In the case of non-linear cost functions until now analytical solutions don't exist, and they will hardly be expected. Thus corresponding investigations should be concentrated on algorithmic solutions and/or various search procedures. One very promising attempt in this direction is the application of evolutionary algorithms. Some basic notions and concepts in accordance to these are summarized in the next section.

3. Evolutionary Optimization: Basic Notions and Concepts

What can we learn from nature? I will give the main answer to this question at first: A general disorder is no chaos, but more likely a special case of order. We need this to understand the working and philosophy of evolution. There must be a functional connection between the state and the parameters of a (natural or artificial) system and its performance.

Here an *evolution strategy* is used for the search for average optimal OD's for the multi-location model with transshipments. To this end it follows now the formulation of the inventory model, described in Section 2, in "evolutionary terms". It is easy to see how the biological objects are in touch with the corresponding mathematical terms. Let

(3) $W = \{ \mathbf{a} \in R^N : 0 \leq a_i < \infty \text{ , } i=1(1)N \}$

denote the parameter space which characterizes the considered system. It contains all OD's $\mathbf{a}$ (*individuals*) of the N-dimensional Euclidean space satisfying the restrictions of all N components a_i (*genes*). Parameter space W is called *population*. A subset $G \subset W$ of OD's denotes a *generation*, i.e., a population at a certain moment. Furthermore, a given value of a gene a_i is called an *allele*. The *objective function*

(4) $g : W \rightarrow R^1$

is defined as a mapping from the parameter space W to the set of real numbers R^1. Value $g(\mathbf{a})$ represents the so called *fitness* of the individual $\mathbf{a}$. The problem now is to find an individual

(5) $\mathbf{a}^* \in W$ with $g(\mathbf{a}^*) \leq g(\mathbf{a})$ for all $\mathbf{a} \in W$,

i.e., an individual with global minimal fitness. However, in general the performance of the system or the fitness of an individual, respectively, can be statistically estimated only by simulation, i.e., only an approximation $g_{Sim}(\mathbf{a},t)$ can be used. Here t means the simulation time, i.e., the time of observing the system. The value of t has a great impact on the accuracy of the approximation from $g_{Sim}(\mathbf{a},t)$ to $g(\mathbf{a})$. If the simulation model is correct it holds

(6) $\lim_{t \rightarrow \infty} g_{Sim}(\mathbf{a},t) = g(\mathbf{a})$ for all $\mathbf{a} \in W$.

The *evolution strategies* were evolved by INGO RECHENBERG in Berlin (see [15]) and further developed by HANS-PAUL SCHWEFEL at the university of Dortmund (see [19] and [20]). These evolutionary algorithms work on a phenotypic level, i.e. they operate directly on the set of real-valued object variables a_i . The actual optimization process carried out by an evolutionary algorithm can be described by an iterative scheme. This scheme is adaptable to specific problems by some variable parameters, but it has to follow always the steps shown in Fig.1. At first we have to choose a starting (or initial) population G_0 by selecting P individuals out of the parameter space W. We can do this by mere chance (uniformly distributed) or, if we know some characteristics of the fitness function, we should take the initial individuals (solutions in the parameter space) from the neighbourhood of the optimum. The example of the Multi-location inventory model with transshipments shows you that we can find this starting population by computing the best stocking levels for stocks without any shipment between

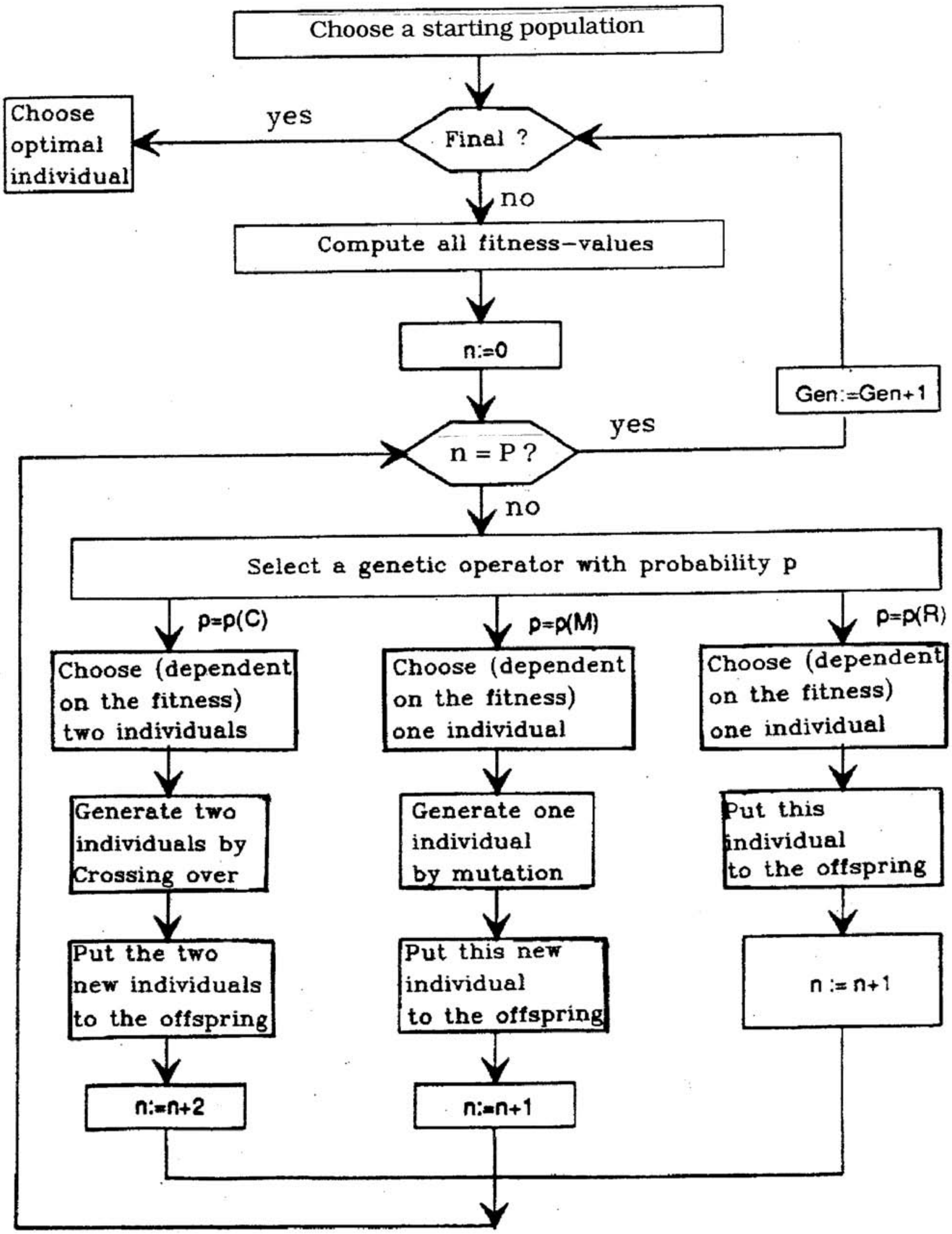

Figure 1. The basic evolutionary algorithm (from [9]).

each other (see [12]). After the starting population is chosen in Fig. 1 follows the "Final?" -question. The nature does not give us any answer to this important question! The natural evolution is a (possibly infinite) continuous process without any recognizable final aim. The reason lies in the sense of evolution: Adaptation of the beings to environmental conditions with permanent change. But the investigated artificial systems have a concrete defined, functional behaviour inside a more or less known "environment". Therefore, the search process should stop after an/the optimum was found. This can be achieved by defining an *epsilon-neighbourhood* around $g(\mathbf{a})$ for $\mathbf{a} \in W$. All fitness-values inside this epsilon-neighbourhood are regarded of equal value. The evolution terminates when there is no essential improvement for the best individual over some generations. Additionally, a maximum number of generations can be set before the evolution process will start.
The good working of evolutionary algorithms depends on well chosen parameters. that means if the offspring (population of children) has fewer individuals than P, we will have to select a genetic operator carefully. The probabilities for the three main operators are:

p(C) for recombination (crossing over)
p(M) for mutation
p(R) for reproduction

Then it holds

$$(7) \qquad p(C) + p(M) + p(R) = 1 .$$

Each of these genetic operators needs one, two or more individuals (OD's) for working. These individuals are chosen from the parent population by a so-called *selection*-procedure. There are a lot of different selection procedures like the roulette, linear ranking, tournament, (N,μ)-selection. Every kind of selection must prefer individuals with a good fitness to such one with a worse. Weak individuals ought to get a though little chance to pass their alleles to the next generation yet. This is very important for the spread out of the individuals over the parameter space. Otherwise the evolution will make a premature convergence to a local optimum, maybe.
The main operator *mutation* is realized by adding to each a_i a normally distributed random number with expected value 0 and standard deviation σ_i. The so-called strategy variables σ_i are stored in an additional vector with length N. Theoretical considerations for a maximum rate of convergence suggest that the optimal settings of the σ_i may depend on the distance from the optimum, i.e., they are a local feature of the response surface (cp. [19]). Therefore, the genetic information of each individual not only consists of the a_i, but also of the strategy parameters σ_i which also undergo mutation and recombination before they are used to mutate the a_i.
The *recombination* is easy made by heredity of an objective variable a_i from one of the two parents, which will be uniform randomly chosen. The child's deviation σ_i is the average of the two parents' deviations.
The performance of evolutionary algorithms depends on well chosen parameters like population size P, number of genes N, number of generations Γ, probabilities of recombination, mutation, and reproduction. There are many different suggestions for the "right" size P of the population. We must face the fact that it is nearly always important to use as small a population as possible, because the total number of fitness evaluations $P \bullet \Gamma$ should be small. This demand is necessary since the time of fitness evaluation for one individual by simulation is much more longer then all the time of creating the next generation. The values stated in literature (P=30 [17], P=50 [18] or P>50 [9]) have proved themselves time and again with many experiments. SCHAFFER suggests in [17] the following formula:

$$(8) \qquad P \approx 1.65 \bullet 2^{\,0.21 \bullet N} .$$

If you look for a good set of parameters to create your own evolution for solving a specific optimization, you will find it by HESSER in [8]. It is very time expensive to get a suitable and acceptable set of parameters, because all parameters influence each other. But fortunately, the evolutionary algorithms are very robust against little changes of their parameters.

4. Some Optimization Results for the Multi-location Inventory Model

A simulation tool was designed in [6] which approximates the inventory costs for given OD over an observation time of maximum 1000 periods. Different demand distributions are available. For the present investigations 500 periods are used only.
The first exemplary inventory model should consist of 4 stocks with the following parameters:

K = (0, 0, 0, 0) ; no order costs
H= (1, 2, 4, 3) ; units per product
P = (10, 9, 11, 8) ; units per product

$$C = \begin{bmatrix} \blacksquare & 5 & 7 & 4 \\ 7 & \blacksquare & 8 & 5 \\ 9 & 8 & \blacksquare & 7 \\ 8 & 7 & 9 & \blacksquare \end{bmatrix} \quad \text{; units per product}$$

Furthermore, we assume a normal distributed demand with a mean of
μ = (200, 300, 250, 150) and deviation σ = (30, 40, 20, 10)
units per period.
Now we have all parameters to start the simulation for a given stocking level. The obvious conclusion is that one individual consists of 4 genes (see [2]).
The evolution process was started with these parameters:

- size of parent population μ: 2
- size of children population Γ: 10
- number of generations G: 250
- rate of crossing over p(C): 0.2
- rate of mutation p(M): 0.7
- rate of reproduction p(R): 0.1 (1 - p(C)-p(M))

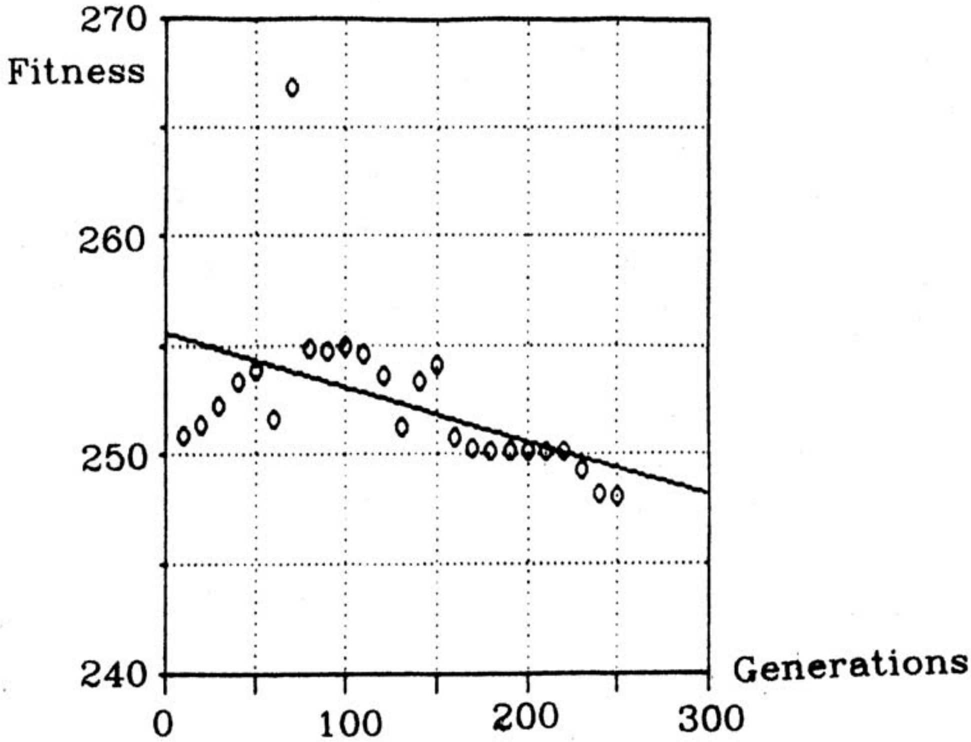

Fig.2: The fitness-value of the best individual in every tenth generation.
The curve was got from 25 fix points by exponential regression:
certainty: 0.25253 correlation coeff: 0.50252 deviation: 0.000159

The rates p(C), p(M) and p(R) will not changed during the evolution takes place. The first individual of the start-population can be created by analytical solving of the problem for fully independently working stocks (see [12]):

$$\mathbf{a}^{(1)} = (240, 336, 262, 156)$$

The second individual of the start population is reproduced simply from $\mathbf{a}^{(1)}$ by mutation. Now we can start the evolution to find the minimum inventory costs and to observe its work.

There are a lot of interesting things recognizable in Fig. 2. At first the individuals spread out from the start-point over the parameter space. The best individuals die because there are too many sub-optimal individuals! I call it as the boy scout phenomenon. The evolution gains implicit knowledge about the structure of the parameter space for itself. Only after a few generations of knowledge acquisition the evolution will start to search for the real optimum.

The optimum was found after 250 generations by

$$\mathbf{a}^{*} = (238.8, 324.1, 256.6, 151.9) \text{ with } g(\mathbf{a}^{*}) = 248.1 \text{ units of inventory costs.}$$

The second exemplary inventory system should consist of maximum 5 locations with the following parameters:

the order cost vector	$K = (0, 0, 0, 0, 0)$,
the holding cost vector	$H = (3, 5, 2, 4, 6)$,
the shortage cost vector	$P = (7, 9, 8, 11, 10)$ and
the transporting costs	$c_{ij} = 5$ for $i, j=1(1)5, i \neq j$.

Furthermore, we assume an exponentially distributed demand with the mean value vector

$$1/\lambda = (200, 300, 500, 400, 250)$$

demand units per period.

Obviously, an individual consists of 5 genes only, each for one location. The evolution process was started with probabilities p(C) and p(M) shown on the left-hand side in Tab.1. These rates will not be changed during the evolution process. The first individual of the starting population can be created in an analytical way by computing the optimal OD for the case of independent locations (see [12]): $\mathbf{a}^{(1)} = (240, 308, 804, 528, 245)$. In the case of independent locations these stocking levels lead to expected average costs of 7159 units of inventory costs.
Tab. 1 shows the results for 9 different combinations of parameter values for the inventory model with 4 or 5 locations, respectively. Both were inquired with a [2/10] and a [4/8] [parents/children] relation. This relation has an important influence on the probability of survival for each individual.

		4 stocks		5 stocks	
p(M)	p(c)	[2/10]	[4/8]	[2/10]	[4/8]
0.1	0.2	3861 - 380	3861 - 60	4398 - 440	4422 - 450
0.1	0.4	3800 - 440	3856 - 440	4426 - 360	4419 - 450
0.1	0.6	3859 - 450	3860 - 110	4422 - 190	4411 - 430
0.3	0.2	3793 - 350	3856 - 120	4413 - 120	4417 - 300
0.3	0.4	3860 - 380	3863 - 140	4417 - 300	4389 - 230
0.3	0.6	3863 - 430	3860 - 340	4414 - 240	4420 - 210
0.5	0.2	3860 - 310	3856 - 260	4416 - 50	4422 - 190
0.5	0.4	3867 - 480	3819 - 360	4436 - 70	4398 - 190
0.7	0.2	3861 - 360	3861 - 90	4425 - 470	4395 - 190

Table 1: Results of the optimization. The left value in each column shows the best cost after 500 generations. The other value is the number of investigated generations until this optimum was found.

The data in Tab.1 allow the following conclusions:

(i) 3860 and 4420 units of money are good approximations for the optimal expected average costs in the 4-location and 5-location case, respectively. Therefore, the costs for the 5-location case decrease to 62 % of the costs which will arise for independent locations.
(ii) The mutation rate p(M) should be greater than the recombination rate p(C). This statement is generally valid for evolution strategies (cp. [19]).
(iii) To give an answer for a good approximation **a** to the optimal OD **a*** is not easy. For instance, the simulation experiments of the 5-location system with a [4/8] parents-to-children relation yield $233 < a_2 < 313$. However, the corresponding estimated average costs are approximately equal. These circumstance lead to the hypothesis that the area around the optimum is very flat. Thus the approximation of g(**a***) is sufficient, even though the approximation a for **a*** has a great variance.

5. Conclusions

The evolutionary optimization needs about 3000 simulation runs, whereas an optimization algorithm special designed for convex functions by HADER and WINKLER in [6] needs only about 100 simulation runs for the problem with 4 locations. But, evolutionary algorithms are suitable for a high performance optimization of very chaotic, completely unknown or non-differentiable objective functions. Thus you can say that the considered inventory problem is not hard enough for an impressing optimization by evolutionary algorithms. But still you can see how these algorithms work, and the observed results support the theoretic assertion that they will find the or a good solution in the most frequent cases.

Future investigations will be concentrated on two directions:

1. The definition of effective evolution parameters for problems like the optimal control of multi-location systems with transshipments and
2. The application of the evolutionary approach to multi-location systems with non-linear cost functions.

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